# HW 3 Solutions

1. This isn't just false, it is stupid. (0+1)* is all strings of 0's and 1's. 0*+1* has strings with only 0's or only 1's.

2. Suppose this language was regular; let n be its pumping constant. Consider the string $w=(^n)^n$. If w =xyz is any decomposition of w satisfying the Pumping Lemma then |xy| <= n, so y consists of a positive number of left parens. Then $xy^2z$ has more left parens than right parens, so it isn't in the language. This violates the Pumping Lemma so the language isn't regular.

3. Same as (2); use string $w=a^nbc^n$ where n is the pumping constant.

4. (a)(b)and (d) are regular; DFAs for each of these are not hard to create. (c) is not regular: if it was let n be the first perfect square larger than the pumping constant and let $w=1^n$. The Pumping Lemma applied to w would say that there is some number k (the length of y) so that n+i*k is always a perfect square. So any two successive perfect squares would differ by k. But perfect squares get farther apart as they get larger: $(x+1)^2-x^2$ is 2x+1, not a constant.

5. Here's a way to build a DFA for L/a. Start with a DFA for L. Make a new DFA with the same states and transitions, but whose final states are all of the states that have a transition on a to a final state in the DFA for L. Then string w gets to a final state in this new automaton if and only if wa gets to a final state in the DFA for L.

6. The pumping constant for a language is the number of states in a DFA for the language. So, given a DFA let n be its number of states. If the automaton accepts any string of length n or more it will accept infinitely many strings. Also, if it accepts a string of length k which is greater than n, it will accept a string that is shorter by at most n letters. That means that if L is regular and infinite it must contain a string of length between n and 2*n. The algorithm is just to check all such strings.

   You can also do (6) in a graph-theoretic way by looking for a cycle in the graph formed by the DFA. However, to do this you need to show that the cycle is reachable from the start state, and that some final state is reachable from the cycle.